Modelling of composite magnetic materials in the quasistatic range

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Abstract. Numerical and physical modelling of heterogeneous magnetic media in the quasistatic range (where eddy currents have to be taken into account) show that an Effective Medium Theory devoted to the description of these materials is pertaining to the behaviour of an effective particle, the size of which is a function of the load in magnetic matter. The particles clustering in the whole concentration range (including the constitution of a conducting path throughout the actual material) seems to be quite well accounted for by this EMT.

1. INTRODUCTION

The modelling of soft heterogeneous magnetic materials is often carried out by using an Effective Medium Theory (EMT) based on Bruggeman's theory [1]. In the case of a two component mixture, where only one is magnetic, the two major parameters which are encountered in the EMT are the effective shape factor and the so-called intrinsic permeability, denoted by N and μ_1 respectively. The expression of the former includes both the shape coefficients of particles and their spatial distributions, therefore N can be viewed, in a sense, as representating the mesostructure of the medium [2]. Experimental studies carried out in the low frequency domain have allowed to identified the latter parameter μ_1 to the rotational permeability $\mu_i=\mu_{stat}$ [3]. The aim of this paper is to show in which way μ_1 could be also relevant to the concentration dependent electrical phenomenons (coming from eddy currents) appearing in the composite medium in the QuasiStatic Range (QSR).

2. THE INTRINSIC PERMEABILITY IN THE QUASISTATIC RANGE

Applying the EMT to the simplest case of magnetic ellipsoids (permeability μ_1, volume fraction C) randomly dispersed in a non magnetic matrix (volume fraction 1-C), and submitted to an uniform magnetic field $\mathbf{H_0}$, leads to the following relation:

$$C.\mathbf{H}_1 + (1-C).\mathbf{H}_2 = \mathbf{H}_0 \qquad (1)$$

where $\mathbf{H_1}$ and $\mathbf{H_2}$ are fictitious fields, given by relation (2) below, and where parameters N and μ_k must be carefully defined:

$$\mathbf{H}_k = \frac{\mu}{\mu + N(\mu_k - \mu)}\mathbf{H}_0 \qquad (2)$$

(This relation is obtained by solving Laplace's equation for an ellipsoidal body (permeability μ_k, k=1,2) immersed in a homogeneous medium (permeability μ) where the field H_0 is supposed to be uniform [4]). To our mind, particles satisfying to this description of an heterogeneous magnetic medium are effective particles, the basic properties of which are strongly correlated to those of randomly dispersed actual particles.

In this way, we have proved experimentaly [2] the following expression in the case ellipsoidal bodies of revolution (permeability μ_1), and with the shape factor N_z along the major axis:

$$N = (2\delta + 3N_z(1 - N_z)) / (6\delta + 3N_z + 3) \qquad (\text{with } \delta = \mu / (\mu_1 - \mu)) \qquad (3)$$

This relation shown that N must be seen as an effective shape factor describing the *mesostructure* of the composite material, rather than a mean shape factor. Similarly, we think that the informations on *the physical properties* of the composite media would be assigned to the permeability μ_1 of the effective particle, the internal field of which is given by the relation (2). In particular, in the QSR, μ_1 would be a function of the frequency f of the applied magnetic field.
Theoretical studies [5] [6] have shown that the complex permeability μ_i of an *isolated conducting particle* (resistivity ρ) *immersed in vacuum* and submitted to an ac-field is reduced by a factor which is a function of the frequency f. The part of this factor (named the softening factor) is to take eddy currents effects into account. References [5] and [6] show that, when the applied field lies along the diameter of the body, the softening parameter of a single particle can be written as:

$$\text{for a sphere:} \quad A_S = 2.\frac{k.a_0.\cos(k.a_0) - \sin(k.a_0)}{\sin(k.a_0) - k.a_0.\cos(k.a_0) + k^2.a_0^2.\sin(k.a_0)} \qquad (4.1)$$

$$\text{for a disk:} \quad A_D = \frac{\dfrac{1}{k.a_0}.\dfrac{J_1(k.a_0)}{J_0(k.a_0)}}{1 - \dfrac{1}{k.a_0}.\dfrac{J_1(k.a_0)}{J_0(k.a_0)}} \qquad (4.2)$$

where J_0 and J_1 are the Bessel functions of the first kind

$$\text{with} \qquad k.a_0 = \sqrt{\frac{2\pi.f.a_0^2.\mu_0|\mu_i|}{\rho}}\, e^{-j\pi/4} e^{j\mathrm{Arg}(\mu_i)/2} \qquad (4.3)$$

and where a_0 is a characteristic length of the conducting body.

Then one could suggest to extend the validity of the EMT to the QSR by setting $\mu_i = \mu_{stat}$ in (4.3) and $\mu_1 = A(f).\mu_{stat}$ in relation (2). (It has been previously underlined that $A_{S,D} \to 1$ when $f \to 0$ and $A_{S,D} \to 0$ when $f \to \infty$: at frequencies high enough one will get $\mu_1 = 0$: the magnetic particle will behave as an ideal diamagnetic body [5]). However, the reliability of this extended EMT obviously depends on the meaning attributed to the length a_0. Indeed, if the assumption introducing the parameter a_0 as the particle radius seems to remain quite appropriate when the magnetic load is small, it is certainly more doubtful when the electrical percolation threshold is reached, namely when aggregated conducting particles constitute infinite conducting paths. In the next sections, we propose to highlight the significance of a_0.

3. FINITE ELEMENT MODELLING OF 2D HETEROGENEOUS MAGNETIC MATERIALS

This part presents simulations, obtained with a finite elements aid, of the magnetic behaviour of 2D heterogeneous materials. This model consists in 312 disks (radius $r_0 = \sqrt{2}.10^{-3}$ m) disposed at the nodes of a square lattice. Each of them can be either non magnetic ($\mu_{stat}=1$) or both magnetic and conducting ($\mu_{stat}=20$, $\rho = 10^{-7}\,\Omega.\mathrm{m}$). At a given concentration in magnetic matter, the magnetic disks are dispersed at random on the lattice sites (figure 1). The electrical contacts between neighbouring conducting disks is assigned to be ideal.This allows eddy currents to flow throughout the medium when the electrical percolation is reached.

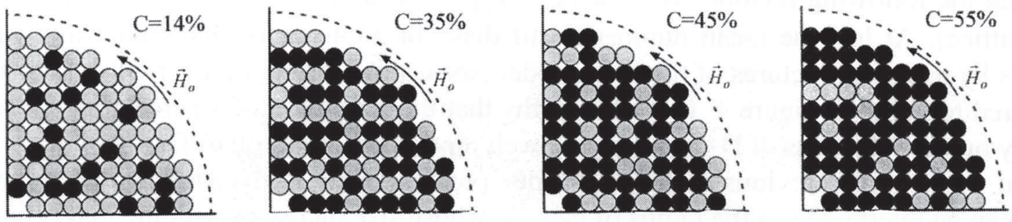

Fig. 1: First quadrant of the 2D model. Particles (magnetics disks are black) are submitted to the applied field H_0. The mean number (D_2) of disks belonging to a cluster, for each surfacic concentrations, $C=0.14$:$D_2=1.14$, $C=0.35$:$D_2=2.79$, $C=0.45$:$D_2=12.5$, $C=0.55$:$D_2=248$, $C=0.70$:$D_2=312$ (not represented). A percolating cluster is built for $0.45<C\leq 0.55$.

The computation of the magnetic flux density throughout the medium leads, for each frequency, to its mean permeability $\mu(f)$, which is taken as the effective permeability of the homogenized medium. We will examined first the static case ($\mu_1=\mu_{stat}$). Agreement with the EMT predictions is good (figure 2). The slight discrepancy observed for C=0.70 between theory (established for an infinite medium) and our simulation can easily be interpreted by the boundary effects [7].

Let us now examine the results got in the QSR. The extraction of the permeability $\mu_i=A(f).\mu_{stat}$ from relation (2) gives, at each concentration and frequency, the value of the softening parameter A(f). The results we got for the real part A'(f) are presented on figure 3.

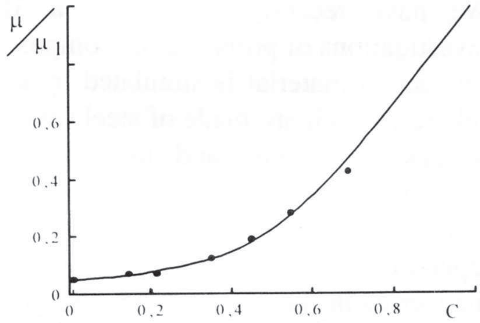

Fig. 2:Validation of the static EMT applied to 2D heterogeneous magnetic materials. Full line is the theory (eq.(1) and (2) with N=1/2 and $\mu_1=20$), the circles are the results of the computation from the 2D model.

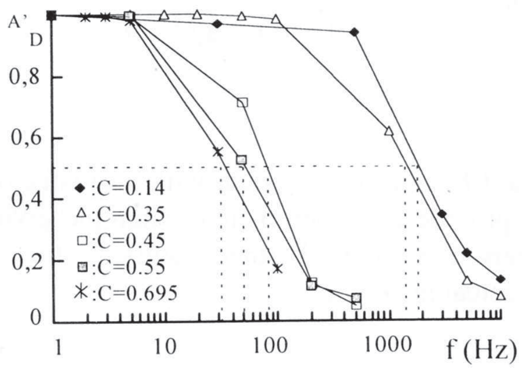

Fig.3: Real part of the softening parameter obtained with the 2D modelling. Note the gap of the typical frequency f_0 defined by $A'_D(f_0)=1/2$. ($C=0.14$:$f_0=1800$Hz),($C=0.35$:$f_0=1300$Hz),($C=0.45$:$f_0=90$Hz),($C=0.55$:$f_0=50$Hz),($C=0.7$:$f_0=30$Hz).

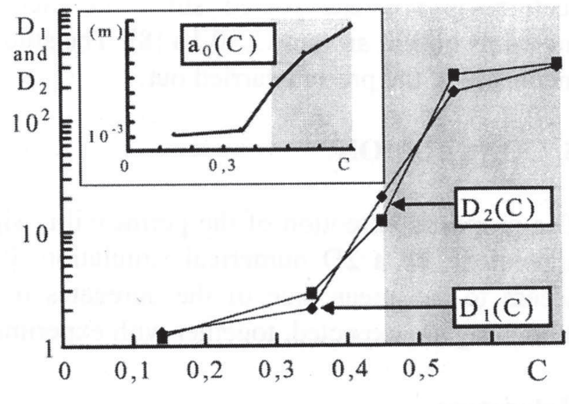

Fig. 4: The mean numbers of disks in an cluster, as calculated by two independent ways: $D_1(C)$ is deduced from the size of the effective particle (see $a_0(C)$ inserted); $D_2(C)$ is directly deduced from the filling of the lattice (see fig.1).

Two groups of curves can be unanbiguously distinguished on figure 3, depending on the load in magnetic matter is situated above or below a particular concentration. This threshold (denoted C_p) occurs just when the disks form a percolating cluster, that is when the concentration belongs to the range 0.45-0.55 (fig. 1). The value of C_p is probably linked to this electrical percolation process, which is expected to occur sharply at C=0.46 (fraction of space occupied by the disks) in the case of a infinite lattice [8], but rather smoothly and starting around C=0.35 when the number of sites of the lattice neared 400. [7]. In the following step, we have assumed for A(f) an expression of the same kind as relation (4.2), but with a

characteristic length depending on the concentration C. We have got for this function $a_0(C)$ the variations inserted in figure 4. Next, we have linked $a_0(C)$ to the mean number D_1 of disks agregated to form a cluster by using the following relation: $D_1 = (a_0(C)/a_0)^2$ (a relation applicable to touching disks placed on a square lattice). At last the mean number D_2 of disks in a cluster is determined in a independent manner, that is by using the pictures of the built model (see caption of the figure 1). Variations of D_1 and D_2 are compared together on figure 4. It is noteworthy that the formation of a percolating cluster , which appears clearly on the variations of D_2, is also quite well reproduced by those of D_1.
Therefore, and opposite to previous remarks [5], our point of view is that the characteristic length in relation (4.1) can be interpreted as the radius of particle whose size increases with the magnetic load. This particle can be thought to be the effective one (not actual) to which the EMT must be applied.

4. MACROSCOPIC MODEL OF 3D HETEROGENEOUS MAGNETIC MATERIALS

We have recently developped a 3D macroscopic model for the investigations of properties of composite materials [9].
The actual material is simulated by some 3000 equally sized touching spheres, which are made of steel (magnetic and with high conductivity) or glass (magnetic and insulating). Their diameter can be either a_1=1mm or a_2=3mm. A toroidal sample of the mixture is placed in a coaxial cell, the detection is provided by a two phases lock-in amplifier. Operating from the measured effective permeability in a identical manner as in the 2D case, we got the softening factors of the effective (spherical) particle as shown figure 5 (the magnetic load was near C=0.10 in the two simulations). By using relation (4.1) the best fits of theses two experimental results are obtained for a parameter a_0 whose values are, in both cases, nearly equal to the radius of spheres. This result, which is in accordance with the conclusions of the previous section, is not unexpected since the electrical percolation in 3D materials occurs around C_p=0.16 [8]. The study of highest concentrated media are at the present carried out.

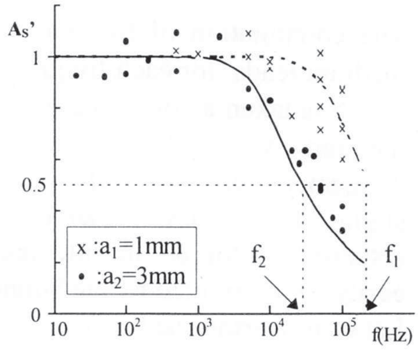

Fig. 5: Real part of the softening parameter, got with the 3D model. The volumic concentrations are near 0.10 in both cases. As predicted by the eq 4.1, the square root of the frequencies for which A_s'=1/2 are in the inverse ratio than the radius of the spheres: $f_1/f_2 \approx (a_2/a_1)^2 = 9$.

5. CONCLUSION

The proposed extention of the permeability significance in the EMT has been tested, with a satisfactory agreement, by a 2D numerical simulation. It could lead to provide an experimental technique giving access to the mean size of the agregates of randomly dispersed particles. Further results on the 3D modelling are expected, together with experimental results on actual materials.

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